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A Study of the  $A$  dependence of deep- inelastic scattering of leptons and its implications for understanding of the EMC effect

### Abstract

It is suggested to determine the  $A$  dependence of distortions of the nucleon structure function  $F_2(x)$  by summing the distortions over an interval  $(x_1, x_2)$ .

It was found from the analysis of data on deep- inelastic scattering of muons and electrons from nuclei that the  $A$  dependence of distortion magnitudes obtained in each of three regions under study, namely shadowing, antishadowing and the *EMC effect* region, follow the same functional form, being different in the normalizing factor only. All the available data give evidence for the saturation of the distortion magnitude with rising  $A$ .

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## 1. Introduction

The investigation of distortions of the nucleon structure function  $F_2(x, Q^2)$  by the nuclear medium, known as the EMC effect, is a subject of activity even after more than ten years since the first observation of the effect from experiments on the deep- inelastic scattering (DIS) of muons on iron and deuterium nuclei. This is clear from recent publications on the study of the  $A$  dependence of DIS of electrons in the E139 – SLAC experiment [1], from reports on the studies of DIS of muons on nuclei from E665 – FNAL experiment [2] and EMC (CERN) experiment [3], as well as from publications on the models developed for the description of nuclear effects over the entire range of Bjorken  $x$  [4, 5]. The *distortions* are defined as a deviation from unity of the ratio  $r^A(x) \equiv F_2^A(x)/F_2^D(x)$ , where  $F_2^A(x)$  and  $F_2^D(x)$  are the structure functions per nucleon measured on a nucleus of mass  $A$  and on a deuteron, respectively.

From our point of view the  $A$  dependence of nuclear effects gets much less attention in theoretical papers than the problem of their  $x$  dependence. To a large extent this is due to the technical difficulties of collecting the data from DIS experiments on a large number of nuclei. For instance, the largest number of different nuclear targets which has been used by SLAC (apart from the deuteron) is eight but for the range of  $x > 0.2$  only (EMC effect region). The data from muon DIS experiments in this kinematic region have poor statistics except for the BCDMS experiment [6], which collected data from two nuclei, N and Fe. In contrast to the E139 – SLAC experiment, muon experiments can detect secondary muons emitted at very small angles (about 2 mrad in the NMC (CERN) experiment), which makes it possible to study nuclear effects in the shadowing ( $x < 0.07$ ) and antishadowing ( $0.07 < x < 0.2$ ) regions.

There exists, however, another reason related to the kind of approach used for the study of the  $A$  dependence, namely, all groups consider data at fixed  $x$ . As a result  $r^A$  either varies little with  $A$  or does not depend on  $A$  near crossover points  $x_I$ ,  $x_{II}$  and  $x_{III}$ , where  $r^A = 1$ . If one fits the  $A$  dependence with the relation

$$r^A = CA^{\alpha(x)}, \quad (1)$$

one obtains [1]  $\alpha(x)$  from  $\sim 0$  to  $-0.04$  in the interval  $0.2 < x < 0.7$  and from  $-0.04$  to  $+0.04$  in the interval  $0.7 < x < 0.9$ . The conclusion of this paper is that for each value of  $x$  the ratio  $\sigma^A/\sigma^D$  (which is equal to the ratio  $F_2^A/F_2^D$ ) decreases approximately logarithmically up to the highest value of  $A$  showing no saturation effects [1]. Similar conclusions follow from results obtained in a different  $x$  range (the region of nuclear shadowing) in the experiments on the DIS of muons on nuclei [3, 7].

Despite the evident importance of those results they are not easy to use in theoretical considerations of the  $A$  dependence because of strong correlations of the parameter  $\alpha(x)$  with the  $x$  dependence of  $r^A$ , which is not yet reproduced by any of the suggested theoretical models. In saying this we of course mean not qualitative agreement which is provided by the majority of models [8] but a quantitative agreement which can be reached in some  $x$  intervals only. One should mention the success of paper [4] in which a good quantitative description of  $r^A(x)$  has been reached for the measurements on  ${}^4\text{He}$  and deuterium nuclei. This result has been obtained by considering nuclear shadowing in the QCD model together with the hypothesis of nucleon stretching in line with a suggestion in ref. [9]. The agreement of the calculations with the data gets worse when one goes to C and Ca nuclei.

To obtain more informative data on the  $A$  dependence it was suggested in ref. [10] to determine the asymptotic behavior of  $\alpha(x)$  which would correspond to the case of infinite nuclear matter:  $A \rightarrow \infty$ . The suggestion does not of course eliminate the correlations. The considered in ref. [10] extrapolation of  $r^A$  as a linear function of  $A^{-1/3}$  to  $A = \infty$  is rather problematic since experimental errors in each  $x$  point do not allow to find possible deviations from linear dependence.

## 2. Defining the structure function distortion magnitude as independent of $x$ and $Q^2$

In contrast to the existing approaches which consider the  $A$  dependence of the DIS of leptons in terms of the ratio  $r^A$  for fixed  $x$  value, in this paper we introduce a conception of the *distortion magnitude*  $\mathcal{M}(A)$  of the structure function determined from deviations of  $r^A$  from unity in some interval  $(x_1, x_2)$ . Then the  $A$  dependence of  $r^A$  is represented as

$$r^A(x, A) = f(x, \mathcal{M}) , \quad (2)$$

where, by the definition, the dimensionless parameter  $\mathcal{M}(A)$  is independent of  $x$  and equals to zero if in the entire interval  $r^A=1$ . This approach exploits the conservation of total nucleon momentum carried by partons, from which it follows that one can not consider the distortion of the nucleon structure function by the nuclear medium (as well as the deviation of  $r^A$  from unity) at some point  $x$  as independent of the distortions observed at the adjacent point  $x + \Delta x$ .

As in the conventional approach we consider structure function distortions as independent of the 4-momentum transfer  $Q^2$  at which  $r^A(x)$  is measured. This is justified by conclusions about the  $Q^2$  independence of  $r^A$  in the range

$0.2 \text{ GeV}^2 < Q^2 < 200 \text{ GeV}^2$  [1, 6, 8].

Our choice of the function  $f(x, \mathcal{M})$  was motivated by the possibility to factorize the  $x$  dependence of  $r^A$  in the range of  $0.003 < x < 0.7$  in accordance with differences in the  $r^A$  behavior found in the three intervals namely the (1) shadowing, (2) antishadowing and (3) EMC effect regions:

$$r^A(x, A) = x^{m_1}(1 + m_2)(1 - m_3x), \quad (3)$$

where the parameters  $m_i$  correspond to the introduced earlier distortion magnitude  $\mathcal{M}(A)$  for each interval.

### 3. Data analysis in the range $0.003 \leq x \leq 0.7$

The parameters  $m_i$  were determined by fitting  $r^A(x)$  measured in the range  $0.003 \leq x \leq 0.7$  on different nuclear targets with eqn.(3). Data obtained from the new generation of DIS experiments with incident muons and electrons which have small statistic and systematic errors were used for the fit along with the EMC data obtained on C and Ca nuclei in the shadowing region [12]. It was required that for each nucleus there should exist data over the entire  $x$  range.

Nucleus	Experiment	Number of Points
He	NMC [11] + SLAC [1]	18 + 14
C	EMC [12] + NMC [11] + SLAC [1]	9 + 18 + 14
Ca	EMC [12] + NMC [11] + SLAC [1]	9 + 18 + 14
Cu	EMC [3]	10

We did not include relative normalization of data as a free parameter although the comparison of the results on  $r^A$  obtained by EMC and NMC in the shadowing region demonstrates a clear systematic shift. We used instead the total experimental error determined by adding statistical and systematic errors at each point in quadrature. For each of four nuclei good agreement ( $\chi^2/d.o.f. \leq 1$  with eqn.(3) has been found.

The results of the fit are given by the solid lines in fig.1. The observed agreement is evidence for a universal form of eqn.(3) for all nuclei and also for the increase of the distortion magnitudes  $m_i$  with  $A$ . The parameters  $m_i$  are shown in fig.2 as a function of  $A$ . The errors for the  $m_3$  values are smaller than the size of the dots in the plot. All three groups of points vary approximately as  $A^{1/3}$ . By fitting the data with

$$m_i = k_i A^{1/3} \quad (4)$$

we find that only  $m_2$  is in good agreement with eqn.(4) :  $\chi^2/d.o.f. = 1.16$ . The data for all three regions indicate possible saturation of the  $A$  dependence for  $A > 20$ . The observed experimental  $A$  dependence is similar to that suggested in ref. [13] for the treatment of structure function distortions within the framework of the three-nucleon correlation model. According to ref. [13] the  $A$  dependence of the EMC effect can be represented by the factor  $\delta(A)$

$$\delta(A) = N \left( 1 - \frac{1}{A^{1/3}} - \frac{1.145}{A^{2/3}} + \frac{0.93}{A} + \frac{0.88}{A^{4/3}} - \frac{0.59}{A^{5/3}} \right), \quad (5)$$

with a normalization constant  $N = 0.27$ . Such an  $A$  dependence was obtained by postulating that the nucleons which reside at the surface of a large nucleus can be excluded from consideration because of the sharply reduced (in accordance with Woods-Saxon potential shape) surface density. The parameter  $\delta(A)$  is shown in fig.2 as a dotted line. Both the data and  $\delta(A)$  tend to saturate at large  $A$ . With a mere change of the normalization constant in eqn.(5) we obtain three lines (solid lines in fig.2) which satisfactorily follow the fitted parameters  $m_i(A)$ .

Thus, all the data considered in this section give evidence for a universal  $A$  dependence of the distortion magnitudes  $k_i \mathcal{M}(A)$  of the nucleon structure function for all three regions, which is satisfactorily described by eqn.(5) confirming the expectation of small effects from the surface nucleons.

#### 4. Data analysis in the range $0.2 \leq x \leq 0.7$

The part of eqn.(3) which correspond to the distortions observed in the EMC effect region coincides (except for the sign of  $m_3$ ) with the linear dependence used in refs. [14, 15, 6] for quantitative estimation of the EMC effect in the interval  $0.2 < x < 0.6$ . The straight line fit was discarded after it had been found that  $r^A$  oscillates around  $r^A=1$  in the region below  $x=0.3$ . Our point is that even this piece of data on  $r^A$  considered alone can yield important information on the  $A$  dependence of the structure function distortion magnitude. With this goal in mind we fitted the data in the interval  $0.2 \leq x \leq 0.7$  with a straight line:

$$r^A(x) = a - bx. \quad (6)$$

We used in the fit the values of  $r^A(x)$  obtained in ref.[1] on He, Be, Al, Ca, Fe, Ag and Au nuclei along with the data of ref.[6] (Fe) and ref.[3] (Cu). The

resulting  $b(A)$  are plotted in fig.3. Figure 4 shows the coordinates of the second crossover point  $x_{II}$  obtained as

$$x_{II}(A) = \frac{a(A) - 1}{b(A)} . \quad (7)$$

The solid line in fig.3 corresponds to eqn.(5) with  $N = 0.54$ . As seen in fig.3, the  $A$  dependence of the distortion magnitude  $b(A)$  is similar to those observed for  $m_i$  plotted in fig.2. In addition one finds in fig.3 a considerable deviation of the data points from the  $CA^{1/3}$  dependence, which is a natural consequence of a diffuse surface of large nuclei. At the same time eqn.(5) satisfactory describes the data up to  $A \sim 64$ . With  $A$  rising further one finds evidence for saturation of the distortion magnitude; the last four points for  $b(A)$  are consistent with  $b(A) = \text{const} = 0.389 \pm 0.012$  ( $\chi^2/\text{d.o.f.} = 0.6$ ). From our analysis and also from the data plotted in fig.4 it follows that  $x_{II} = \text{const}$  and equals  $0.273 \pm 0.010$ .<sup>1</sup>

This result considered with the increase of the distortion magnitude  $b(A)$  (oscillation amplitude) of the nucleon structure function observed in fig.3 gives evidence for the simultaneous increase of distortions, with  $A$  from  $A=4$  on, in the regions  $x < x_{II}$  and  $x > x_{II}$ , which is in agreement with the conclusion on the similarity of the  $A$  dependence of  $m_i$ .

The effect of the  $A$  independence of the  $x_{II}$  coordinate can be used for the study of how parton distributions are distorted by nuclear medium. This follows from the well known property of the valence and sea quarks distributions which reach a maximum on each side of the  $x_{II}$  point. Thus, if the parton distribution of one sort only were distorted (e.g. that of valence quarks), the coordinate  $x_{II}$  would have changed its value with rising  $A$ .

## 5. $x$ Dependence of $F_2^{A_1}/F_2^{A_2}$

In this section we make use of eqn.(3) and of the results on the  $A$  dependence obtained in sections **3** and **4** to determine the  $x$  dependence of the ratio of structure functions measured on  $A_1$  and  $A_2$  nuclei with  $A > 2$ .

The form of eqn.(3), the similarity of the  $A$  dependence for  $m_i$  and also the saturation observed for  $b(A)$  in the  $A > 20$  region allows one to infer that:

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<sup>1</sup>We do not discuss the  $A$  dependence of the two other crossover points  $x_I$  and  $x_{III}$ , where one needs both higher statistical accuracy and larger number of nuclei to be able to draw quantitative conclusions

(a) in the region  $x \ll 1$

$$F_2^{A_1}/F_2^{A_2} = C_1 x^\alpha, \quad (8)$$

where

$$\begin{aligned} \alpha &= m_1(A_1) - m_1(A_2), \\ C_1 &= \frac{1 + m_2(A_1)}{1 + m_2(A_2)}; \end{aligned}$$

(b) in the range of  $x > 0.1$  and for  $A_1 \approx A_2$  or  $A_1, A_2 > 20$  the following relation holds:

$$F_2^{A_1}/F_2^{A_2} \approx 1. \quad (9)$$

We have determined  $\alpha^{Ca/C}$  by using  $m_1$  values found for C and Ca in section 3 and also  $\alpha^{Ca/Li}$  by interpolating  $m_1(A)$ . The ratios  $F_2^{Ca}/F_2^C$  and  $F_2^{Ca}/F_2^{Li}$  determined with eqn.(8) are shown in fig.5 with solid lines superimposed on experimental data obtained in ref.[16]. The hatched area is the uncertainty in  $\alpha^{Ca/Li}$  which comes from the error in  $m_1(Ca)$  and from the interpolation uncertainty for the parameter  $m_1(Li)$ . The data shown in fig.5a favors the conclusions summarized in eqns.(8)–(9), while the data in fig.5b indicate a possible discrepancy between eqn.(8) and the experiment.

## 6. Conclusions

In conclusion, from the analysis of the data on the structure functions ratio  $F_2^A(x)/F_2^D(x)$  in the framework suggested in this paper, we have obtained the magnitude of the integral distortions of the nucleon structure function in the shadowing, antishadowing and the EMC effect regions as a function of atomic mass  $A$ . The similarity of the  $A$  dependence found in all the three regions makes it possible to predict the distortion effects in the range  $x \ll 1$  by using the results obtained in the EMC effect region.

Good agreement is obtained between the data and the function  $r^A(x, A)$  defined by eqn.(3) and an explicit form for the  $A$  dependence of the distortion magnitude  $\mathcal{M}(A)$ . The position of the second crossover point  $x_{II}$  is found to be independent of  $A$ . From these facts, we can draw the following conclusions:

- 1) The distortions of nucleon structure functions (or of parton distributions), that show up as characteristic oscillations of  $r^A(x)$  around unity are due to the transition from  $A=2$  to  $A=4$ ; <sup>2</sup>

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<sup>2</sup>We can not make any statement on the case of a transition from  $A=2$  to  $A=3$  due to lack of data on  $F_2^{^3He}(x)/F_2^D(x)$

2) With the increase from  $A=4$  to  $\sim 20$ , the shape of the  $x$  dependence of  $r^A(x)$  does not change. The distortion magnitudes (oscillation amplitudes) increase monotonically with  $A$  in good agreement with the parameter  $\delta(A)$  [13] (which serves the purpose of excluding the effect of surface nucleons). This increase is similar in all three regions, and reveals a common  $A$  dependence;

3) For  $A > 20$  the distortion magnitudes are virtually  $A$  independent;

4) The ratios of the structure functions, measured on nuclei  $A_1$  and  $A_2$  with atomic mass  $A > 2$ , are described in the region  $x \ll 1$  by a simple formula  $C_1 x^\alpha$ , while for the region  $x > 0.1$  for nuclei which have nearly equal mass numbers and also for  $A > 20$ , the following relation holds:  $F_2^{A_1}/F_2^{A_2} \approx 1$ .

The conclusions 1) — 3) give evidence for a unique property of the 4-nucleon system as the system responsible for the distortions of the nucleon structure function. The problem of understanding the structure of a nucleon bound in a nucleus is thus split into explanations of two phenomena: (a) how the oscillations turn on for the ratio of  $F_2^{He}/F_2^D$  and (b) how the distortion magnitude  $\mathcal{M}(A)$  (oscillation amplitude) is amplified with the increase of  $A$ .

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*Fig.1.* The results of the fit with eqn. (3) of the  $F_2^A/F_2^D$  measured on He [1, 11], C [1, 11, 12], Ca [1, 11, 12] and Cu [3]

*Fig.2.* The nucleon structure function distortion magnitudes  $m_i$  versus  $A$  determined in the regions of nuclear shadowing ( $i=1$ ), antishadowing ( $i=2$ ) and EMC effect ( $i=3$ ). The dotted line corresponds to the factor  $\delta(A)$  [13], and the solid lines were obtained by multiplying  $\delta(A)$  with a normalization factor, different for each  $x$  – range. The results of the fit with  $k_i A^{1/3}$  are shown with dashed lines.

*Fig.3.* The nucleon structure function distortion magnitude versus  $A$  determined in the range  $0.2 \leq x \leq 0.7$  with a straight line fit of  $r^A(x)$ . The solid line corresponds to the factor  $\delta(A)$  [13] with normalization  $N=0.54$ . The dashed line shows the result of the fit of the first four points of  $b(A)$  with  $kA^{1/3}$

*Fig.4.* The coordinates  $x_{II}$  which correspond to crossing of the straight line  $r^A = a - bx$  with  $r^A = 1$ .



(a)



(b)

Fig.5. The results of the measurement [16] of  $F_2^{Ca}/F_2^C$  — (a) and  $F_2^{Ca}/F_2^{Li}$  — (b) compared with  $C_1 x^\alpha$ , where  $\alpha$  was found from analysis of the  $F_2^A/F_2^D$  ratios









